MATIBIA UMIVERSITY
OF SCIEMCE AMD TECHMOLOGY

## FACULTY OF ENGINEERING AND SPATIAL SCIENCE DEPARTMENT OF MINING AND PROCESS ENGINEERING

| QUALIFICATION : BACHELORS OF ENGINEERING IN MINING ENGINEERING |  |
| :--- | :--- |
| QUALIFICATION CODE: BEMIN | LEVEL: 6 |
| COURSE CODE: GSS710S | COURSE NAME: GEOSTATISTICS |
| SESSION: JUNE 2022 | PAPER: THEORY |
| DURATION: 3 HOURS | MARKS: 100 |


| SECOND OPPORTUNITY EXAM |  |
| :--- | :--- |
| EXAMINER(S) | Mallikarjun Rao Pillalamarry |
| MODERATOR: | Lawrence Madziwa |

## INSTRUCTIONS

1. Answer all questions.
2. Read all the questions carefully before answering.
3. Marks for each question are indicated at the end of each question.
4. Please ensure that your writing is legible, neat and presentable.

## PERMISSIBLE MATERIALS

1. Examination paper.

## This EXAM has two sections. Section A and B.

## Time allowed: 3 hours

## SECTION A

## Instructions: Answer any 2 questions. Excess questions will not be marked.

Question 1 Answer the following questions as succinctly as possible
a) Discuss the revenue factors involved in operating a mining venture
b) Discuss the main factors involved in the valuation of an ore body
c) Distinguish the main differences between Geostatistics and statistics. Discuss the differences that are apparent in a data set with a statistical and a geostatistical variance. Explain the effects of these two variances on a mine scenario where block grades are being evaluated.

Question 2
a) What are the limitations of statistical data in solving geological and mining problems of grade-tonnage relationships?
b) Discuss the following modes of exploration indicating what happens and what steps follow afterwards: Geochemistry, Geological and Geophysics.

## Question 3

a) Discuss the differences between resources and reserves
b) What are the purposes of ore reserves evaluation?
c) What is regression effect and how can it be overcome?
d) What is data optimisation? In what ways can data be optimised?
e) What are simulations and how do we use them on a mine setting?

## SECTION B

Instructions: Answer Question 1 and any 2 other questions. Excess questions will not be marked.
Question 1 is compulsory.

## Question 1

a) What is support?
b) Which statistical quantity represents reliability of an estimation method?
c) What is the necessary condition for a variogram model to be used in Kriging estimation method?
d) What is auxiliary function $\mathrm{F}(1, \mathrm{~b})$ gives?
e) What ' $D$ ' matrix in Kriging system of equations represents?
f) Which of spherical, gaussian and exponential variograms is more continues at the origin?
g) What is the difference between extension variance and Kriging variance?
h) What is the difference between zonal anisotropy and geometric anisotropy?
i) What is screen effect in Kriging? [1]

Question 2
a) Briefly discuss the effects of scale, nugget effect and range on Kriging weights
b) Spatial continuity of Zinc grades in the orebody is following spherical variogram model
with a sill value of $2.5(\%)^{2}$ and range of 300 m . Determine the model values for the given lags.

$$
\gamma(h)=\left\{\begin{array}{l}
C\left[1.5 \frac{h}{a}-\frac{1}{2}\left(\frac{h}{a}\right)^{3}\right] \text { ifh } \leq \mathrm{a} \\
C \quad \text { otherwise }
\end{array}\right.
$$

$\begin{array}{llllll}\text { Lags: } & 50 & 100 & 200 & 300 & 400\end{array}$

Question 3 In a copper deposit, to excavate a stope having a size of $30 \mathrm{~m} \times 30 \mathrm{~m}$, two level of 15 m apart were driven as shown in Figure 1. The average grade of level 1 was found to be $5.4 \%$ and level 2 was $6.7 \%$. If the grade of the stope is taken as average grade of both levels, estimate the extension variance. Spatial continuity in the deposit is best described with spherical varigoram having range of influence of 90 m and a sill of $0.6(\%)^{2}$


Figure 1
Question 4 Copper grade at three borehole locations A, B, and C are found to be $7.8 \%, 5.2 \%$ and $6.2 \%$ respectively. Grade at a location X to be estimated. The inter distance between ( $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and X ) is given in the form of a distance matrix. Estimate the grade at location ' X ' with 96\% confidence using Kriging method of estimation?

$$
\gamma(h)=1.5\left[1-\exp \left(-\frac{h}{150}\right)\right][\text { Variogram model }]
$$

Distance Matrix

|  | A | B | C | X |
| :---: | :---: | :---: | :---: | :---: |
| A | 0 | 29.2 | 53.9 | 26.9 |
| B | 29.2 | 0 | 25.5 | 50.2 |
| C | 53.9 | 25.5 | 0 | 75.7 |
| X | 26.9 | 50.2 | 75.7 | 0 |

$\mathrm{C}^{-1}$
$\left[\begin{array}{cccc}-1.9 & 1.71 & 0.19 & 0.46 \\ 1.71 & -3.67 & 1.96 & 0.09 \\ 0.19 & 1.96 & -2.15 & 0.45 \\ 0.46 & 0.09 & 0.45 & -0.23\end{array}\right]$

## Standard Normal Probabilities



Table entry for $z$ is the area under the standard normal curve to the left of $z$.

| $z$ | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -3.4 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0002 |
| -3.3 | . 0005 | . 0005 | . 0005 | . 0004 | . 0004 | . 0004 | . 0004 | . 0004 | . 0004 | . 0003 |
| -3.2 | . 0007 | . 0007 | . 0006 | . 0006 | . 0006 | . 0006 | . 0006 | . 0005 | . 0005 | . 0005 |
| -3.1 | . 0010 | . 0009 | . 0009 | . 0009 | . 0008 | . 0008 | . 0008 | . 0008 | . 0007 | . 0007 |
| -3.0 | . 0013 | . 0013 | . 0013 | . 0012 | . 0012 | . 0011 | . 0011 | . 0011 | . 0010 | . 0010 |
| -2.9 | . 0019 | . 0018 | . 0018 | . 0017 | . 0016 | . 0016 | . 0015 | . 0015 | . 0014 | . 0014 |
| -2.8 | . 0026 | . 0025 | . 0024 | . 0023 | . 0023 | . 0022 | . 0021 | . 0021 | . 0020 | . 0019 |
| -2.7 | . 0035 | . 0034 | . 0033 | . 0032 | . 0031 | . 0030 | . 0029 | . 0028 | . 0027 | . 0026 |
| -2.6 | . 0047 | . 0045 | . 0044 | . 0043 | . 0041 | . 0040 | . 0039 | . 0038 | . 0037 | . 0036 |
| -2.5 | . 0062 | . 0060 | . 0059 | . 0057 | . 0055 | . 0054 | . 0052 | . 0051 | . 0049 | . 0048 |
| -2.4 | . 0082 | . 0080 | . 0078 | . 0075 | . 0073 | . 0071 | . 0069 | . 0068 | . 0066 | . 0064 |
| -2.3 | . 0107 | . 0104 | . 0102 | . 0099 | . 0096 | . 0094 | . 0091 | . 0089 | . 0087 | . 0084 |
| -2.2 | . 0139 | . 0136 | . 0132 | . 0129 | . 0125 | . 0122 | . 0119 | . 0116 | . 0113 | . 0110 |
| -2.1 | . 0179 | . 0174 | . 0170 | . 0166 | . 0162 | . 0158 | . 0154 | . 0150 | . 0146 | . 0143 |
| -2.0 | . 0228 | . 0222 | . 0217 | . 0212 | . 0207 | . 0202 | . 0197 | . 0192 | . 0188 | . 0183 |
| -1.9 | . 0287 | . 0281 | . 0274 | . 0268 | . 0262 | . 0256 | . 0250 | . 0244 | . 0239 | . 0233 |
| -1.8 | . 0359 | . 0351 | . 0344 | . 0336 | . 0329 | . 0322 | . 0314 | . 0307 | . 0301 | . 0294 |
| -1.7 | . 0446 | . 0436 | . 0427 | . 0418 | . 0409 | . 0401 | . 0392 | . 0384 | . 0375 | . 0367 |
| -1.6 | . 0548 | . 0537 | . 0526 | . 0516 | . 0505 | . 0495 | . 0485 | . 0475 | . 0465 | . 0455 |
| -1.5 | . 0668 | . 0655 | . 0643 | . 0630 | . 0618 | . 0606 | . 0594 | . 0582 | . 0571 | . 0559 |
| -1.4 | . 0808 | . 0793 | . 0778 | . 0764 | . 0749 | . 0735 | . 0721 | . 0708 | . 0694 | . 0681 |
| -1.3 | . 0968 | . 0951 | . 0934 | . 0918 | . 0901 | . 0885 | . 0869 | . 0853 | . 0838 | . 0823 |
| -1.2 | . 1151 | . 1131 | . 1112 | . 1093 | . 1075 | . 1056 | . 1038 | . 1020 | . 1003 | . 0985 |
| -1.1 | . 1357 | . 1335 | . 1314 | . 1292 | . 1271 | . 1251 | . 1230 | . 1210 | . 1190 | . 1170 |
| -1.0 | . 1587 | . 1562 | . 1539 | . 1515 | . 1492 | . 1469 | . 1446 | . 1423 | . 1401 | . 1379 |
| -0.9 | . 1841 | . 1814 | . 1788 | . 1762 | . 1736 | . 1711 | . 1685 | . 1660 | . 1635 | . 1611 |
| -0.8 | . 2119 | . 2090 | . 2061 | . 2033 | . 2005 | . 1977 | . 1949 | . 1922 | . 1894 | . 1867 |
| -0.7 | . 2420 | . 2389 | . 2358 | . 2327 | . 2296 | . 2266 | . 2236 | . 2206 | . 2177 | . 2148 |
| -0.6 | . 2743 | . 2709 | . 2676 | . 2643 | . 2611 | . 2578 | . 2546 | . 2514 | . 2483 | . 2451 |
| -0.5 | . 3085 | . 3050 | . 3015 | . 2981 | . 2946 | . 2912 | . 2877 | . 2843 | . 2810 | . 2776 |
| -0.4 | . 3446 | . 3409 | . 3372 | . 3336 | . 3300 | . 3264 | . 3228 | . 3192 | . 3156 | . 3121 |
| -0.3 | . 3821 | . 3783 | . 3745 | . 3707 | . 3669 | . 3632 | . 3594 | . 3557 | . 3520 | . 3483 |
| -0.2 | . 4207 | . 4168 | . 4129 | . 4090 | . 4052 | . 4013 | . 3974 | . 3936 | . 3897 | . 3859 |
| -0.1 | . 4602 | . 4562 | . 4522 | . 4483 | . 4443 | . 4404 | . 4364 | . 4325 | . 4286 | . 4247 |
| -0.0 | . 5000 | . 4960 | . 4920 | . 4880 | . 4840 | . 4801 | . 4761 | . 4721 | . 4681 | . 4641 |

## Standard Normal Probabilities



Table entry for $z$ is the area under the standard normal curve to the left of $z$.

| $z$ | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | . 5000 | . 5040 | . 5080 | . 5120 | . 5160 | . 5199 | . 5239 | . 5279 | . 5319 | . 5359 |
| 0.1 | . 5398 | . 5438 | . 5478 | . 5517 | . 5557 | . 5596 | . 5636 | . 5675 | . 5714 | . 5753 |
| 0.2 | . 5793 | . 5832 | . 5871 | . 5910 | . 5948 | . 5987 | . 6026 | . 6064 | . 6103 | . 6141 |
| 0.3 | . 6179 | . 6217 | . 6255 | . 6293 | . 6331 | . 6368 | . 6406 | . 6443 | . 6480 | . 6517 |
| 0.4 | . 6554 | . 6591 | . 6628 | . 6664 | . 6700 | . 6736 | . 6772 | . 6808 | . 6844 | . 6879 |
| 0.5 | . 6915 | . 6950 | . 6985 | . 7019 | . 7054 | . 7088 | . 7123 | . 7157 | . 7190 | . 7224 |
| 0.6 | . 7257 | . 7291 | . 7324 | . 7357 | . 7389 | . 7422 | . 7454 | . 7486 | . 7517 | . 7549 |
| 0.7 | . 7580 | . 7611 | . 7642 | . 7673 | . 7704 | . 7734 | . 7764 | . 7794 | . 7823 | . 7852 |
| 0.8 | . 7881 | . 7910 | . 7939 | . 7967 | . 7995 | . 8023 | . 8051 | . 8078 | . 8106 | . 8133 |
| 0.9 | . 8159 | . 8186 | . 8212 | . 8238 | . 8264 | . 8289 | . 8315 | . 8340 | . 8365 | . 8389 |
| 1.0 | . 8413 | . 8438 | . 8461 | . 8485 | . 8508 | . 8531 | . 8554 | . 8577 | . 8599 | . 8621 |
| 1.1 | . 8643 | . 8665 | . 8686 | . 8708 | . 8729 | . 8749 | . 8770 | . 8790 | . 8810 | . 8830 |
| 1.2 | . 8849 | . 8869 | . 8888 | . 8907 | . 8925 | . 8944 | . 8962 | . 8980 | . 8997 | . 9015 |
| 1.3 | . 9032 | . 9049 | . 9066 | . 9082 | . 9099 | . 9115 | . 9131 | . 9147 | . 9162 | . 9177 |
| 1.4 | . 9192 | . 9207 | . 9222 | . 9236 | . 9251 | . 9265 | . 9279 | . 9292 | . 9306 | . 9319 |
| 1.5 | . 9332 | . 9345 | . 9357 | . 9370 | . 9382 | . 9394 | . 9406 | . 9418 | . 9429 | . 9441 |
| 1.6 | . 9452 | . 9463 | . 9474 | . 9484 | . 9495 | . 9505 | . 9515 | . 9525 | . 9535 | . 9545 |
| 1.7 | . 9554 | . 9564 | . 9573 | . 9582 | . 9591 | . 9599 | . 9608 | . 9616 | . 9625 | . 9633 |
| 1.8 | . 9641 | . 9649 | . 9656 | . 9664 | . 9671 | . 9678 | . 9686 | . 9693 | . 9699 | . 9706 |
| 1.9 | . 9713 | . 9719 | . 9726 | . 9732 | . 9738 | . 9744 | . 9750 | . 9756 | . 9761 | . 9767 |
| 2.0 | . 9772 | . 9778 | . 9783 | . 9788 | . 9793 | . 9798 | . 9803 | . 9808 | . 9812 | . 9817 |
| 2.1 | . 9821 | . 9826 | . 9830 | . 9834 | . 9838 | . 9842 | . 9846 | . 9850 | . 9854 | . 9857 |
| 2.2 | . 9861 | . 9864 | . 9868 | . 9871 | . 9875 | . 9878 | . 9881 | . 9884 | . 9887 | . 9890 |
| 2.3 | . 9893 | . 9896 | . 9898 | . 9901 | . 9904 | . 9906 | . 9909 | . 9911 | . 9913 | . 9916 |
| 2.4 | . 9918 | . 9920 | . 9922 | . 9925 | . 9927 | . 9929 | . 9931 | . 9932 | . 9934 | . 9936 |
| 2.5 | . 9938 | . 9940 | . 9941 | . 9943 | . 9945 | . 9946 | . 9948 | . 9949 | . 9951 | . 9952 |
| 2.6 | . 9953 | . 9955 | . 9956 | . 9957 | . 9959 | . 9960 | . 9961 | . 9962 | . 9963 | . 9964 |
| 2.7 | . 9965 | . 9966 | . 9967 | . 9968 | . 9969 | . 9970 | . 9971 | . 9972 | . 9973 | . 9974 |
| 2.8 | . 9974 | . 9975 | . 9976 | . 9977 | . 9977 | . 9978 | . 9979 | . 9979 | . 9980 | . 9981 |
| 2.9 | . 9981 | . 9982 | . 9982 | . 9983 | . 9984 | . 9984 | . 9985 | . 9985 | . 9986 | . 9986 |
| 3.0 | . 9987 | . 9987 | . 9987 | . 9988 | . 9988 | . 9989 | . 9989 | . 9989 | . 9990 | . 9990 |
| 3.1 | . 9990 | . 9991 | . 9991 | . 9991 | . 9992 | . 9992 | . 9992 | . 9992 | . 9993 | . 9993 |
| 3.2 | . 9993 | . 9993 | . 9994 | . 9994 | . 9994 | . 9994 | . 9994 | . 9995 | . 9995 | . 9995 |
| 3.3 | . 9995 | . 9995 | . 9995 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9997 |
| 3.4 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9998 |

Auxiliary function $\gamma(\mathrm{L}, \mathrm{B})$ for Spherical model with range 1.0 and sill 1.0

|  | $B$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L$ | .1 | .2 | .3 | .4 | .5 | .6 | .7 | .8 | .9 |  |


|  | $B$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L$ | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 5.0 |
| .05 | .524 | .575 | .617 | .652 | .681 | .737 | .777 | .806 | .828 | .861 |
| .10 | .545 | .594 | .634 | .667 | .695 | .748 | .786 | .814 | .836 | .867 |
| .15 | .573 | .619 | .656 | .687 | .714 | .764 | .799 | .825 | .846 | .875 |
| .20 | .605 | .648 | .682 | .711 | .735 | .782 | .814 | .838 | .857 | .884 |
| .25 | .641 | .679 | .711 | .737 | .759 | .801 | .831 | .853 | .870 | .894 |
| .30 | .678 | .712 | .741 | .764 | .784 | .822 | .848 | .868 | .883 | .905 |
| .35 | .715 | .746 | .771 | .792 | .809 | .843 | .866 | .884 | .897 | .917 |
| .40 | .753 | .780 | .801 | .820 | .835 | .864 | .884 | .899 | .911 | .928 |
| .45 | .790 | .812 | .831 | .847 | .860 | .884 | .902 | .915 | .924 | .939 |
| .50 | .825 | .844 | .860 | .872 | .883 | .904 | .918 | .929 | .937 | .949 |
| .55 | .858 | .873 | .886 | .897 | .906 | .922 | .934 | .943 | .949 | .959 |
| .60 | .888 | .901 | .911 | .919 | .926 | .939 | .948 | .955 | .960 | .968 |
| .65 | .915 | .925 | .933 | .939 | .944 | .954 | .961 | .966 | .970 | .976 |
| .70 | .939 | .946 | .952 | .956 | .960 | .967 | .972 | .976 | .979 | .983 |
| .75 | .959 | .964 | .968 | .971 | .974 | .978 | .982 | .984 | .986 | .989 |
| .80 | .975 | .978 | .981 | .983 | .984 | .987 | .989 | .991 | .992 | .993 |
| .85 | .987 | .989 | .990 | .991 | .992 | .993 | .994 | .995 | .996 | .997 |
| .90 | .995 | .996 | .996 | .997 | .997 | .997 | .998 | .998 | .998 | .999 |
| .95 | .999 | .999 | .999 | .999 | .999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 1.00 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

Auxiliary function $\mathrm{F}(\mathrm{L} ; \mathrm{B})$ for Spherical model with range 1.0 and sill 1.0

|  | $B$ |  |  |  |  |  |  |  |  |  |  | B |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L$ | . 1 | . 2 | . 3 | . 4 | . 5 | . 6 | . 7 | . 8 | . 9 | 1.0 | $L$ | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 5.0 |
| . 10 | . 078 | . 120 | . 165 | . 211 | . 256 | . 300 | . 342 | . 383 | . 422 | . 457 | . 10 | . 520 | . 572 | . 614 | . 650 | . 679 | . 735 | . 775 | . 804 | . 827 | . 860 |
| . 20 | . 120 | . 155 | . 196 | . 237 | . 280 | . 321 | . 362 | . 401 | . 438 | . 473 | . 20 | . 534 | . 584 | . 625 | . 659 | . 688 | . 743 | . 781 | . 810 | . 832 | . 864 |
| . 30 | . 165 | . 196 | . 231 | . 270 | . 309 | . 349 | . 387 | . 424 | . 460 | . 493 | . 30 | . 5.51 | . 600 | . 639 | . 672 | . 700 | . 752 | . 789 | . 817 | . 838 | . 869 |
| . 40 | . 211 | . 237 | . 270 | . 305 | . 342 | . 379 | . 415 | . 451 | . 484 | . 516 | . 40 | . 572 | . 618 | . 655 | . 687 | . 713 | . 763 | . 799 | . 825 | . 845 | . 874 |
| . 50 | . 256 | . 280 | . 309 | . 342 | . 376 | . 411 | . 445 | . 479 | . 511 | . 541 | . 50 | . 593 | . 637 | . 673 | . 703 | . 728 | . 775 | . 809 | . 834 | . 853 | . 881 |
| . 60 | . 300 | . 321 | . 349 | . 379 | . 411 | 443 | . 476 | . 507 | . 538 | . 566 | . 60 | . 616 | . 657 | . 691 | . 719 | . 743 | . 788 | . 820 | . 843 | . 861 | . 887 |
| . 70 | . 342 | . 362 | . 387 | . 415 | . 445 | . 476 | . 506 | . 536 | . 565 | . 591 | . 70 | . 638 | . 677 | . 709 | . 736 | . 758 | . 800 | . 830 | . 852 | . 870 | .894 |
| . 80 | . 383 | . 401 | . 424 | . 451 | . 479 | . 507 | . 336 | . 564 | . 591 | . 616 | . 80 | . 660 | . 697 | . 727 | . 752 | . 773 | . 813 | . 841 | . 861 | . 878 | . 901 |
| . 90 | . 422 | . 438 | . 460 | . 484 | . 511 | . 538 | . 565 | . 591 | . 616 | . 640 | . 90 | . 682 | . 716 | . 744 | . 767 | . 787 | . 824 | . 851 | . 870 | . 885 | . 907 |
| 1.00 | . 457 | . 473 | . 493 | . 516 | . 541 | . 566 | . 591 | . 616 | . 640 | . 662 | 1.00 | . 701 | . 733 | . 760 | . 782 | . 800 | . 835 | . 860 | . 878 | . 892 | . 913 |
| 1.20 | . 520 | . 534 | . 551 | . 572 | . 593 | . 616 | . 638 | . 660 | . 682 | . 701 | 1.20 | . 736 | . 764 | . 788 | . 807 | . 823 | . 854 | . 876 | . 892 | . 905 | . 923 |
| 1.40 | . 572 | . 584 | . 600 | . 618 | . 637 | . 657 | . 677 | . 697 | . 716 | . 733 | 1.40 | . 764 | . 790 | . 811 | . 828 | . 842 | . 870 | . 890 | . 904 | . 915 | . 931 |
| 1.60 | . 614 | . 625 | . 639 | . 655 | . 673 | . 691 | . 709 | . 727 | . 744 | . 760 | 1.60 | . 788 | . 811 | . 829 | . 845 | . 858 | . 883 | . 901 | . 914 | . 924 | . 938 |
| 1.80 | . 650 | . 659 | . 672 | . 687 | . 703 | . 719 | . 736 | . 752 | . 767 | . 782 | 1.80 | . 807 | . 828 | . 845 | . 859 | . 871 | . 894 | . 910 | . 921 | . 931 | . 944 |
| 2.00 | . 679 | . 688 | . 700 | . 713 | . 728 | . 743 | . 758 | . 773 | . 787 | . 800 | 2.00 | . 823 | . 842 | . 858 | . 871 | . 882 | . 903 | . 917 | . 928 | . 936 | . 948 |
| 2.50 | . 735 | . 743 | . 752 | . 763 | . 775 | . 788 | . 800 | . 813 | . 824 | . 835 | 2.50 | . 854 | . 870 | . 883 | . 894 | . 903 | . 920 | . 932 | . 941 | . 948 | . 957 |
| 3.00 | . 775 | . 781 | . 789 | . 799 | . 809 | . 820 | . 830 | . 841 | . 851 | . 860 | 3.00 | . 876 | . 890 | . 901 | . 910 | . 917 | .932 | 942 | . 950 | . 955 | . 964 |
| 3.50 | . 804 | . 810 | . 817 | . 825 | . 834 | . 843 | . 852 | . 861 | . 870 | . 878 | 3.50 | . 892 | . 904 | . 914 | . 921 | . 928 | . 941 | . 950 | . 956 | . 961 | . 969 |
| 4.00 | . 827 | . 832 | . 838 | . 845 | . 853 | . 861 | . 870 | . 878 | . 885 | . 892 | 4.00 | . 905 | . 915 | . 924 | . 931 | . 936 | . 948 | . 955 | . 961 | . 966 | . 972 |
| 5.00 | . 860 | . 864 | . 869 | . 874 | . 881 | . 887 | . 894 | . 901 | . 907 | . 913 | 5.00 | . 923 | . 931 | . 938 | . 944 | . 948 | . 957 | . 964 | . 969 | . 972 | . 977 |

Auxiliary function $\mathrm{H}(\mathrm{L}, \mathrm{B})$ for Spherical model with range 1.0 and sill 1.0

|  | $B$ |  |  |  |  |  |  |  |  |  |  | $B$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L$ | . 1 | . 2 | . 3 | . 4 | . 5 | . 6 | . 7 | . 8 | . 9 | . 10 | $L$ | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 5.0 |
| . 10 | . 114 | . 177 | . 243 | . 310 | . 374 | . 436 | . 494 | . 546 | . 593 | . 633 | . 10 | . 694 | . 738 | . 771 | . 796 | . 817 | . 853 | . 878 | . 895 | . 908 | . 927 |
| . 20 | . 177 | . 227 | . 285 | . 346 | . 406 | . 464 | . 518 | . 568 | . 613 | . 651 | . 20 | . 709 | . 751 | . 782 | . 806 | . 826 | . 860 | . 884 | . 900 | . 913 | . 930 |
| . 30 | . 243 | . 285 | . 336 | . 390 | . 445 | . 499 | . 550 | . 597 | . 639 | . 674 | . 30 | . 729 | . 767 | . 797 | . 819 | . 837 | . 870 | . 891 | . 907 | . 919 | . 935 |
| . 40 | . 310 | . 346 | . 390 | . 439 | . 489 | . 539 | . 586 | . 629 | . 668 | . 701 | . 40 | . 751 | . 786 | . 813 | . 834 | . 850 | . 880 | . 900 | . 914 | . 925 | . 940 |
| . 50 | . 374 | . 406 | . 445 | . 489 | . 535 | . 580 | . 623 | . 663 | . 698 | . 728 | . 50 | . 774 | . 806 | . 830 | . 849 | . 864 | . 891 | . 909 | . 922 | . 932 | . 946 |
| . 60 | . 436 | . 464 | . 499 | . 539 | . 580 | . 621 | . 660 | . 697 | . 728 | . 755 | . 60 | . 796 | . 825 | . 847 | . 864 | . 878 | . 902 | . 918 | . 930 | . 939 | . 951 |
| . 70 | . 494 | . 518 | . 550 | . 586 | . 623 | . 660 | . 696 | . 729 | . 757 | . 781 | . 70 | . 818 | . 844 | . 863 | . 879 | . 891 | . 913 | . 927 | . 938 | . 945 | . 956 |
| . 80 | . 546 | . 568 | . 597 | . 629 | . 663 | . 697 | . 729 | . 758 | . 783 | . 805 | . 80 | . 837 | . 861 | . 878 | . 892 | . 902 | . 222 | . 935 | . 944 | . 951 | . 961 |
| . 90 | . 593 | . 613 | . 639 | . 668 | . 698 | . 728 | . 757 | . 783 | . 806 | . 826 | . 90 | . 855 | . 875 | . 891 | . 903 | . 913 | . 930 | . 942 | . 950 | . 956 | . 965 |
| 1.00 | . 633 | . 651 | . 674 | . 701 | . 728 | . 755 | . 781 | . 805 | . 826 | . 843 | 1.00 | . 869 | . 888 | . 902 | . 913 | . 921 | . 937 | . 948 | . 955 | . 961 | . 969 |
| 1.20 | . 694 | . 709 | . 729 | . 751 | . 774 | . 796 | . 818 | . 837 | . 855 | . 869 | 1.20 | . 891 | . 907 | . 918 | . 927 | . 935 | . 948 | . 956 | . 963 | . 967 | . 974 |
| 1.40 | . 738 | . 751 | . 767 | . 786 | . 806 | . 825 | . 844 | . 861 | .875 | . 888 | 1.40 | . 907 | . 920 | . 930 | . 938 | . 944 | . 955 | . 963 | . 968 | . 972 | . 978 |
| 1.60 | . 771 | . 782 | . 7.97 | . 813 | . 830 | . 847 | . 863 | . 878 | . 891 | . 902 | 1.60 | . 918 | . 930 | . 939 | . 945 | . 951 | . 961 | . 967 | . 972 | . 975 | . 980 |
| 1.80 | . 796 | . 806 | . 819 | . 834 | . 849 | . 864 | . 879 | . 892 | . 903 | . 913 | 1.80 | . 227 | . 938 | . 945 | . 952 | . 956 | . 965 | . 971 | . 975 | . 978 | . 983 |
| 2.00 | . 817 | . 826 | . 837 | . 850 | . 864 | . 878 | . 891 | . 902 | . 913 | . 921 | 2.00 | . 935 | . 944 | . 951 | . 956 | . 961 | . 969 | . 974 | . 978 | . 980 | . 984 |
| 2.50 | . 853 | . 860 | . 870 | . 880 | . 891 | . 902 | . 913 | . 922 | . 930 | . 937 | 2.50 | . 948 | . 955 | . 961 | . 965 | . 969 | . 975 | . 979 | . 982 | . 984 | . 987 |
| 3.00 | . 878 | . 884 | . 891 | . 900 | . 909 | . 918 | . 927 | . 935 | . 942 | . 948 | 3.00 | . 956 | . 963 | . 967 | . 971 | . 974 | . 979 | . 983 | . 985 | . 987 | . 990 |
| 3.50 | . 895 | . 900 | . 907 | . 914 | . 922 | . 930 | . 938 | . 944 | . 950 | . 955 | 3.50 | . 963 | . 968 | . 972 | . 975 | . 978 | . 982 | . 985 | . 987 | . 989 | . 991 |
| 4.00 | . 908 | . 913 | . 919 | . 225 | . 932 | . 939 | . 945 | . 951 | . 956 | . 961 | 4.00 | . 967 | . 972 | . 975 | . 978 | . 980 | . 984 | . 987 | . 989 | . 990 | . 992 |
| 5.00 | . 927 | . 930 | . 935 | . 940 | . 946 | . 951 | . 956 | . 961 | . 965 | . 969 | 5.00 | . 974 | . 978 | . 980 | . 983 | . 984 | . 987 | . 990 | . 991 | . 992 | . 994 |

Auxiliary function $\mathrm{F}(\mathrm{L} ; \mathrm{L} ; \mathrm{B})$ for Spherical model with range 1.0 and sill 1.0

|  | $B$ |  |  |  |  |  |  |  |  |  |  | $B$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  | $L$ | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 5.0 |
| $L$ | . 1 | . 2 | . 3 | 4 | . 5 | . 6 | . 7 | . 8 | . 9 | 1.0 | L 10 | 1.2 | . 577 | . 619 | . 653 | . 683 | . 738 | . 777 | . 807 | . 829 | . 861 |
| . 10 | . 099 | . 136 | . 178 | . 222 | . 266 | . 309 | . 350 | . 390 | 428 | . 464 | 20 | . 550 | . 598 | . 638 | . 671 | . 699 | . 751 | . 789 | . 816 | . 837 | . 868 |
| . 20 | . 168 | . 196 | . 231 | . 269 | . 308 | . 347 | . 385 | . 423 | . 458 | . 491 | . 30 | . 581 | . 626 | . 663 | . 693 | . 719 | . 768 | . 803 | . 829 | . 849 | . 877 |
| . 30 | . 239 | . 262 | . 291 | . 324 | . 358 | . 394 | . 429 | . 463 | . 496 | . 527 | . 40 | . 616 | . 657 | . 691 | . 719 | . 742 | . 787 | . 819 | . 843 | . 861 | . 887 |
| . 40 | . 311 | . 329 | . 353 | . 382 | . 413 | . 445 | . 476 | . 508 | . 538 | . 566 | . 50 | . 6.62 | . 689 | . 720 | . 745 | . 767 | . 808 | . 836 | . 858 | . 874 | . 898 |
| . 50 | . 380 | . 395 | . 416 | . 441 | . 468 | . 497 | . 526 | . 554 | . 581 | . 607 | . 60 | . 688 | . 722 | . 749 | . 772 | . 791 | . 828 | . 854 | . 873 | . 887 | . 909 |
| . 60 | . 445 | . 459 | . 477 | . 499 | . 523 | . 549 | . 574 | . 600 | . 624 | . 648 | . 70 | . 723 | . 753 | . 777 | . 798 | . 815 | . 847 | . 870 | . 887 | . 900 | . 919 |
| . 70 | . 507 | . 519 | . 535 | . 5.54 | . 576 | . 508 | . 622 | . 644 | . 666 | . 687 | . 80 | . 756 |  |  |  | . 837 | . 865 | . 886 | . 901 | . 912 | . 929 |
| . 80 | . 565 | . 574 | . 588 | . 606 | . 625 | . 645 | . 666 | . 686 | . 705 | . 724 | . 80 | . 785 | . 782 | . 828 | . 842 |  |  |  |  |  | . 037 |
| . 90 | . 616 | . 625 | . 637 | . 652 | . 669 | . 687 | . 706 | . 724 | . 741 | . 757 | . 90 | . 785 | . 809 | . 828 | . 843 | . 857 | . 882 | . 900 | . 913 | . 923 | . 937 |
| 1.00 | . 662 | . 669 | . 680 | . 694 | . 709 | . 725 | . 741 | . 757 | . 772 | . 786 | 1.00 | . 811 | . 832 | . 849 | . 862 | . 874 | . 896 | . 912 | . 923 | . 932 | . 945 |
| 1.20 | . 735 | . 741 | . 750 | . 760 | . 772 | . 785 | . 797 | . 810 | . 822 | . 833 | 1.20 | . 853 | . 869 | . 882 | . 893 | . 902 | . 919 | . 931 | . 940 | . 947 | . 957 |
| 1.40 | . 789 | .794 | . 800 | . 809 | . 818 | . 828 | . 839 | . 849 | . 858 | . 867 | 1.40 | . 883 | . 896 | . 906 | . 915 | . 922 | . 936 | . 945 | . 952 | . 958 | . 966 |
| 1.60 | . 828 | . 832 | . 838 | . 845 | . 852 | . 861 | . 869 | . 877 | . 885 | . 892 | 1.60 | . 905 | . 915 | . 924 | . 931 | . 937 | . 948 | . 956 | . 961 | . 966 | . 972 |
| 1.80 | . 858 | . 861 | . 866 | . 872 | . 878 | . 885 | .892 | . 899 | . 905 | . 911 | 1.80 | . 922 | . 930 | . 937 | . 943 | . 948 | . 957 | . 963 | . 968 | . 972 | . 977 |
| 2.00 | . 880 | . 883 | . 887 | . 892 | . 897 | . 903 | . 909 | . 915 | . 920 | . 925 | 2.00 | . 934 | . 941 | . 947 | . 952 | . 956 | . 964 | . 969 | . 973 | . 976 | . 981 |
| 2.50 | . 918 | . 920 | . 923 | . 926 | . 930 | . 934 | . 938 | . 942 | . 946 | . 949 | 2.50 | . 955 | . 960 | . 964 | . 967 | . 970 | . 975 | . 979 | . 982 | . 984 | . 987 |
| 3.00 | . 940 | . 941 | . 944 | . 946 | . 949 | . 952 | . 955 | . 958 | . 960 | . 96.3 | 3.00 | . 967 | . 971 | . 974 | . 976 | . 978 | . 982 | . 985 | . 987 | . 988 | . 991 |
| 3.50 | . 954 | . 955 | . 957 | . 959 | . 961 | . 963 | . 966 | . 968 | . 970 | . 972 | 3.50 | . 975 | . 978 | . 980 | . 982 | . 983 | . 986 | . 988 | . 990 | . 991 | . 993 |
| 4.00 | . 963 | . 964 | . 965 | . 967 | . 969 | . 970 | . 972 | . 974 | . 976 | . 977 | 4.00 | . 980 | . 982 | . 984 | . 986 | . 987 | . 989 | . 991 | . 992 | . 993 | . 994 |
| 5.00 | . 974 | . 975 | . 976 | . 978 | . 979 | . 980 | . 981 | . 983 | . 984 | . 985 | 5.00 | . 987 | . 988 | . 989 | . 990 | . 991 | . 993 | . 994 | . 995 | . 995 | . 996 |

Auxiliary function $\chi(\mathrm{L} ; \mathrm{B})$ for Spherical model with range 1.0 and sill 1.0

|  | $B$ |  |  |  |  |  |  |  |  |  |  | B |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L$ | . 1 | . 2 | . 3 | 4 | . 5 | . 6 | . 7 | . 8 | . 9 | 1.0 | $L$ | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 5.0 |
| . 10 | . 098 | . 136 | . 178 | . 222 | . 266 | . 309 | . 350 | . 390 | . 428 | . 464 | . 10 | . 526 | . 577 | . 619 | . 653 | . 683 | . 738 | . 777 | . 807 | . 829 | . 861 |
| . 20 | . 164 | . 194 | . 229 | . 268 | . 307 | . 346 | . 385 | . 422 | . 458 | . 491 | . 20 | . 550 | . 598 | . 638 | . 671 | . 698 | . 751 | . 788 | . 816 | . 837 | . 868 |
| . 30 | . 233 | . 257 | . 288 | . 321 | . 356 | . 392 | . 427 | . 462 | .495 | . 526 | . 30 | . 580 | . 625 | . 662 | . 693 | . 719 | . 768 | . 803 | . 828 | . 848 | . 877 |
| . 40 | . 302 | . 322 | . 348 | . 378 | . 409 | . 441 | . 474 | . 505 | . 535 | . 564 | . 40 | . 614 | . 655 | . 689 | . 718 | . 741 | . 787 | . 819 | . 842 | . 861 | . 887 |
| . 50 | . 368 | . 385 | . 408 | . 434 | . 462 | . 492 | . 521 | . 550 | . 577 | . 603 | . 50 | . 649 | . 687 | . 718 | . 743 | . 765 | . 806 | . 835 | . 857 | . 873 | . 897 |
| . 60 | . 430 | . 445 | . 466 | . 489 | . 515 | . 541 | . 568 | . 594 | . 619 | . 642 | . 60 | . 684 | . 718 | . 746 | . 769 | . 788 | . 825 | . 852 | . 871 | . 886 | . 907 |
| . 70 | . 488 | . 502 | . 520 | . 541 | . 564 | . 588 | . 612 | . 6336 | . 658 | .680 | . 70 | . 717 | . 747 | . 772 | . 793 | . 811 | . 844 | . 867 | . 885 | . 898 | . 917 |
| . 80 | . 542 | . 554 | . 570 | . 589 | . 610 | . 631 | . 653 | .674 | . 695 | . 714 | . 80 | . 747 | . 774 | . 797 | . 815 | . 831 | . 861 | . 881 | . 897 | . 909 | . 926 |
| . 90 | . 589 | . 600 | . 614 | . 632 | .650 | . 670 | . 689 | . 708 | . 727 | . 744 | . 90 | . 774 | . 798 | . 818 | .835 | . 849 | . 875 | . 894 | . 908 | . 919 | . 934 |
| 1.00 | . 629 | . 639 | . 653 | . 668 | . 685 | . 703 | . 720 | . 737 | . 754 | . 769 | 1.00 | . 796 | . 818 | . 836 | . 851 | . 864 | . 888 | . 905 | . 917 | . 927 | . 941 |
| 1.20 | . 691 | . 699 | . 711 | . 723 | . 737 | .752 | . 767 | . 781 | . 795 | . 808 | 1.20 | . 830 | . 848 | . 864 | . 876 | . 886 | . 906 | . 920 | . 931 | . 939 | . 950 |
| 1.40 | . 735 | . 742 | . 752 | . 763 | . 775 | . 788 | . 800 | . 812 | . 824 | . 835 | 1.40 | . 854 | . 870 | . 883 | . 894 | . 903 | .920 | . 932 | . 941 | . 948 | . 958 |
| 1.60 | . 768 | . 775 | . 783 | . 793 | . 803 | . 814 | . 825 | . 836 | . 846 | . 856 | 1.60 | . 873 | . 886 | . 898 | . 907 | . 915 | . 930 | . 940 | . 948 | . 954 | . 963 |
| 1.80 | . 794 | . 800 | . 807 | . 816 | . 825 | .835 | . 845 | . 854 | . 863 | . 872 | 1.80 | . 887 | . 899 | . 909 | . 917 | . 924 | . 938 | . 947 | . 954 | . 959 | . 967 |
| 2.00 | . 815 | . 820 | . 826 | . 834 | . 842 | . 851 | . 860 | . 869 | . 877 | .885 | 2.00 | . 898 | . 909 | . 918 | . 926 | . 932 | . 944 | . 952 | . 959 | . 963 | . 970 |
| 2.50 | . 852 | . 856 | . 861 | . 867 | . 874 | . 881 | . 888 | . 895 | . 902 | . 908 | 2.50 | . 918 | . 927 | . 934 | . 940 | . 946 | . 955 | . 962 | . 967 | . 971 | . 976 |
| 3.00 | . 876 | . 880 | . 884 | . 889 | . 895 | . 901 | . 907 | . 912 | . 918 | . 923 | 3.00 | .932 | . 939 | . 945 | . 950 | . 955 | . 963 | . 968 | . 972 | . 976 | . 980 |
| 3.50 | . 894 | . 897 | . 901 | . 905 | . 910 | . 915 | . 920 | . 925 | . 930 | . 934 | 3.50 | . 942 | . 948 | . 953 | . 957 | . 961 | . 968 | . 973 | . 976 | . 979 | . 983 |
| 4.00 | . 907 | . 910 | . 913 | . 917 | . 921 | . 926 | . 930 | . 934 | . 938 | . 942 | 4.00 | . 949 | . 955 | .959 | . 963 | . 966 | . 972 | . 976 | . 979 | . 982 | . 985 |
| 5.00 | . 926 | . 928 | . 931 | . 934 | . 937 | . 941 | . 944 | . 947 | . 951 | . 954 | 5.00 | . 959 | . 964 | . 967 | . 970 | . 973 | . 978 | . 981 | . 983 | . 985 | . 988 |

Regularised semi-variogram $\gamma(\mathrm{h})$ for Spherical model with range a and sill 1.0 for various distances h

|  | $h / L$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a / L$ | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
| .50 | .300 | .325 | .325 | .325 | .325 | .325 | .325 | .325 | .325 | .325 |
| 1.00 | .450 | .550 | .550 | .550 | .550 | .550 | .550 | .550 | .550 | .550 |
| 1.50 | .463 | .678 | .681 | .681 | .681 | .681 | .681 | .681 | .681 | .681 |
| 2.00 | .412 | .728 | .756 | .756 | .756 | .756 | .756 | .756 | .756 | .756 |
| 2.50 | .355 | .717 | .802 | .803 | .803 | .803 | .803 | .803 | .803 | .803 |
| 3.00 | .307 | .669 | .822 | .835 | .835 | .835 | .835 | .835 | .835 | .835 |
| 3.50 | .269 | .610 | .812 | .858 | 858 | .858 | .858 | .858 | .858 | .858 |
| 4.00 | .239 | .555 | .778 | .868 | .876 | .876 | .876 | .876 | .876 | .876 |
| 4.50 | .215 | .507 | .733 | .861 | .889 | .889 | .889 | .889 | .889 | .889 |
| 5.00 | .194 | .464 | .686 | .836 | .896 | .900 | .900 | .900 | .900 | .900 |
| 5.50 | .178 | .428 | .642 | .802 | .890 | .909 | .909 | .909 | .909 | .909 |
| 6.00 | .163 | .396 | .601 | .764 | .872 | .914 | .917 | .917 | .917 | .917 |
| 6.50 | .151 | .368 | .564 | .726 | .845 | .909 | .923 | .923 | .923 | .923 |
| 7.00 | .141 | .344 | .530 | .690 | .814 | .895 | .926 | .929 | .929 | .929 |
| 7.50 | .132 | .323 | .500 | .655 | .782 | .874 | .923 | .933 | .933 | .933 |
| 8.00 | .124 | .304 | .472 | .623 | .751 | .849 | .912 | .936 | .938 | .938 |
| 8.50 | .117 | .287 | .447 | .593 | .720 | .822 | .894 | .933 | .941 | .941 |
| 9.00 | .110 | .272 | .425 | .566 | .690 | .794 | .874 | .924 | .943 | .945 |
| 9.50 | .104 | .258 | .404 | .541 | .663 | .767 | .851 | .910 | .941 | .947 |
| 10.00 | .099 | .246 | .386 | .517 | .636 | .741 | .827 | .892 | .933 | .949 |

## Additional Information (GSS710S)

Spherical Variogram Model

$$
\begin{aligned}
\gamma(h) & =C\left[\frac{3}{2}\left(\frac{h}{a}\right)-\frac{1}{2}\left(\frac{h}{a}\right)^{3}\right] & & \text { for } \mathrm{h}<a \\
& =C & & \text { for } \mathrm{h} \geq a
\end{aligned}
$$

Relationship between sill of regularized and point variogram

$$
\begin{aligned}
& C_{L}=C\left[1-\frac{L}{2 a}+\frac{L^{3}}{20 a^{3}}\right] \text { for } \mathrm{L}<a \\
& C_{L}=\frac{C a}{L}\left[\frac{15}{20}-\frac{4}{20} \frac{a}{L}\right] \text { for } \mathrm{L} \geq a
\end{aligned}
$$

Auxiliary functions for Spherical variogram

$$
\begin{gathered}
=\frac{C}{8} \frac{l}{a}\left(6-\frac{l^{2}}{a^{2}}\right) \text { when } l \leq a \\
=\frac{C}{8}\left(8-3 \frac{a}{l}\right) \text { when } l>a \\
=\frac{C}{20} \frac{l}{a}\left(10-\frac{l^{2}}{a^{2}}\right) \quad \text { when } l \leq a \\
=
\end{gathered}
$$


$H(1 ; b)$



